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# Ranking of Octagonal Fuzzy Numbers for Solving Fuzzy Job Sequencing Problem Using Robust Ranking Technique

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**ABSTRACT**
Sequencing problem is to select the order in which the vital tasks are to be done to minimize the total elapsed time taken for all the tasks. Generally, in job sequencing problems, the processing times are precisely valued. But in reality, it is observed that the processing times during the performance of the job are indefinite. For that reason, the conception of fuzzy job sequencing problem make available for use an effective outline which is useful for real-life situations with fuzzy processing times. Here, we suggest a new technique for solution of fuzzy sequencing problems involving Octagonal Fuzz Numbers (OFNs).

In this paper, we propose a simple approach for the solution of fuzzy sequencing problem under fuzzy environment where processing time taken as octagonal fuzzy numbers. It can be solved using robust ranking method and fuzzy sequencing problem can be converted into a crisp valued sequence problem and solved by Johnson's algorithm which is illustrated through a numerical example.

**KEYWORDS**
Ranking, Fuzzy sequencing problem, Octagonal fuzzy numbers, Robust ranking technique.

## 1. Introduction

Optimization is a central concept in operations research, and it involves finding the best possible solution to a problem that satisfies given constraints. Optimization techniques are used in a wide range of fields, including engineering, finance, transportation, logistics, and manufacturing.

Ranking fuzzy number is used in decision-making process in an economic environment. In an organization various activities such as planning, execution, and other process takes place continuously. This requires careful observation of various parameters which are all in uncertain in nature due the competitive business environment globally. In fuzzy environment ranking fuzzy numbers is a very important decision making

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procedure. The idea of fuzzy set was first proposed by Bellman et al. (1970)[1], as a mean of handling uncertainty that is due to imprecision rather than randomness.

Fuzzy systems have gained more and more attention from researchers and practitioners of various fields. In such systems, the output represented by a fuzzy set may need to be transformed into a scalar value, and this task is known as the defuzzification process. Unlike in the case of real numbers, fuzzy quantities have no natural order. As a consequence there are several ranking methods for fuzzy numbers, which suffers some defects in some form, for example, lack of uniqueness. Thus there is no single method that orders the fuzzy numbers correctly. Based on the context of the application, some methods seem to be more appropriate than others. Ranking fuzzy numbers has attracted special research attention due to their wide applications in the theory of fuzzy decision making, risk analysis, data analysis, optimization, etc. In order to achieve greater efficiencies and accuracies in ranking results, several ranking methods have been investigated by many authors since 1976. To cite a few Chu et al. (2002)[4], Deng et al.(2006)[5], Abbasbandy et al.(2010)[2] , Liu et al. (2009)[6], and Amit Kumar et al. (2010)[3].

A job sequencing problems arise when we are concerned with situations where there is a choice in which a number of tasks can be performed. Job sequencing problem has become the major problem in the computer field. Sequencing problem is considered to be one of the classic and important applications of Operations Research (OR). The main role of the classical sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the total amount of time required to complete the process of all the jobs. One of the renowned work in the field of considered till date is by Johnson's who gave the algorithm in 1954 for production scheduling in which he had minimized the total idle time of machines and the total production times of the jobs.

### *1.1. Preliminaries*

In this section, we introduce some fundamental definitions have been investigated of fuzzy numbers.

**Definition 1.1.** If $X$ is a collection of objects denoted generically by $x$, then the fuzzy set $A$ in $X$ is defined to be a set of ordered pairs, proposed by Bellman et al. (1970)[1], and Zadeh (1965)[7]. $\bar{A} = \{(x, \mu_A)(X) \mid x \in X\}$. Where $\mu_A(X)$ is called the membership function for the fuzzy set $\bar{A}$. The membership function maps each element of $x$ to a membership grade or membership value between 0 and 1.

**Definition 1.2.** A Fuzzy number $\tilde{A}$ in the real line $\mathbb{R}$ is a fuzzy set $\mu_{\tilde{A}}(X) : \mathbb{R} \to (0, 1)$ that satisfies the following characteristics: Selvakumari et al., (2017)[10].

(1) There exists an at least one $x \in \mathbb{R}$ with $\mu_{\tilde{A}}(X) = 1$.
(2) $\mu_{\tilde{A}}(X)$ is piece wise continuous.

**Remark 1.** Membership function $\mu_{\tilde{A}}(X)$ are continuous functions

**Definition 1.3.** A fuzzy number is the normal octagonal fuzzy number is denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(X)$ is given by:

$$\mu_{\bar{A}}(X) = \begin{cases} 0 & for\ x < a_1; \\ k\left[\frac{x-a_1}{a_2-a_1}\right] & for\ a_1 \le x \le a_2; \\ k & for\ a_2 \le x \le a_3; \\ k + (1-k)\frac{(x-a_3)}{(a_4-a_3)} & for\ a_3 \le x \le a_4; \\ 1 & for\ a_4 \le x \le a_5; \\ k + (1-k)\frac{(a_6-x)}{(a_6-a_5)} & for\ a_5 \le x \le a_6; \\ k & for\ a_6 \le x \le a_7; \\ k\frac{(a_8-x)}{(a_8-a_7)} & for\ a_7 \le x \le a_8; \\ 0 & for\ x \ge a_8; \end{cases}$$

Where $0 \le k \le 1$

**Remark 2.** If $k = 0$ , the octagonal fuzzy number reduces to trapezoidal fuzzy number $(a_3, a_4, a_5, a_6)$ and if $k = 1$ reduces to trapezoidal fuzzy number $(a_1, a_4, a_5, a_8)$ .

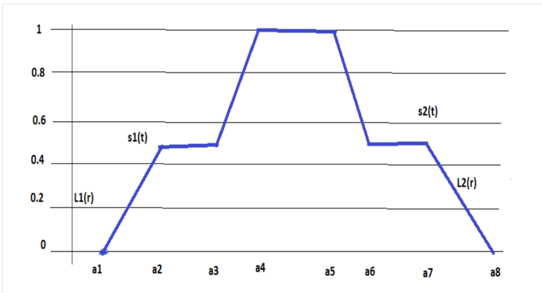


**Figure 1.** Graphical representation of a Octagonal Fuzzy Numbers (OFNs)

## 2. Robust Ranking Technique

To provide results which are consistent with human intuition, robust ranking technique is used and its satisfies compensation, linearity and additive properties. If $a$ is a convex fuzzy number, the robust ranking index is defined by:

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)d\alpha \qquad (1)$$

Where $(a_\alpha^L, a_\alpha^U) = [\{(b-a)\alpha + a, d-(d-c)\alpha\}, \{(f-e)\alpha + e, h-(h-g)\alpha\}]$ is the $\alpha$-level cut of a fuzzy number $\tilde{a}$ . Here this method is proposed for ranking the objective values. The representative value of fuzzy number $\tilde{a}$ is given by robust ranking index $R(\tilde{a})$.

## 3. Mathematical Formulation of $n$ Jobs through $m$ Machines

Let there are $n$ jobs say $A_1, A_2, \ldots, A_n$ be processed through $m$ machines say $M_1, M_2, \ldots, M_n$ in the order $M_1, M_2, \ldots, M_n$. Let $t_{ij}$ be the fuzzy processing time taken by $i^{th}$ job to be completed by $j^{th}$ machine. The mathematical model of the given problem can be stated as: Kripa et al. [8], and V.S.Jadhav et al. (2019)[9].

| Jobs | Machine($M_1$) | Machine($M_2$) | Machine($M_3$) |
|---|---|---|---|
| 1 | $A_{11}$ | $A_{12}$ | $A_{13}$ |
| 2 | $A_{21}$ | $A_{22}$ | $A_{23}$ |
| 3 | $A_{31}$ | $A_{32}$ | $A_{33}$ |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| $n$ | $A_{n1}$ | $A_{n2}$ | $A_{n3}$ |

The well-known Johnsons Method can be extended to this problem, forming two fictitious machines G and H (say), If either (or) both of the following conditions are satisfied.

$$\min(t_{i1}) \geq \max(t_{ij}), \quad j = 2, 3, \ldots, m-1$$
$$\min_i(t_{im}) \geq \max_i(t_{ij}), \quad i = 1, 2, \ldots, n$$

Let G and H are given by $G_i = \sum_{j=1}^{m-1} t_{ij}$ and $H_i = \sum_{j=2}^{m} t_{ij},\ i = 1, 2, \ldots n$ Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time using robust ranking technique.

## 4. Proposed Method for Solving Fuzzy Sequencing Problem

In this study, we proposed a new solving method for fuzzy sequencing problem by using robust ranking technique. The proposed method must operate the following steps:

**Step (1)** : Using robust ranking technique, the fuzzy sequencing problem can be converted into crisp sequencing problem.

**Step (2)** : The optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.

**Step (3)** : After finding the optimal sequence, determine the total elapsed fuzzy time and also the fuzzy idle time on machines.

## 5. Numerical Example

In this section, a numerical example has been considered to illustrate the proposed solution procedure. Consider the following fuzzy sequencing problem.

Global export house has to process 5 items through 3 stages of production viz. Cutting, Sewing & Pressing. Processing times are given in the following table.

| **Items** | **Cutting** $M_1$ | **Sewing** $M_2$ | **Pressing** $M_3$ |
|---|---|---|---|
| $A_1$ | $(-2,-1,0,1,4,5,6,7)$ | $(-3,-2,-1,0,1,2,3,4)$ | $(7,8,10,11,14,15,17,18)$ |
| $A_2$ | $(1,2,3,4,5,6,9,10)$ | $(-3,-2,-1,0,4,5,6,7)$ | $(6,8,9,10,12,13,14,16)$ |
| $A_2$ | $(1,2,4,5,8,10,12,14)$ | $(-4,-3,0,1,2,3,4,5)$ | $(-1,0,1,2,4,5,6,7)$ |
| $A_4$ | $(2,3,6,7,8,10,11,13)$ | $(-2,-1,0,1,2,3,4,5)$ | $(5,6,7,8,12,13,14,15)$ |
| $A_5$ | $(3,5,6,8,10,12,13,15)$ | $(-1,0,1,2,3,4,5,6)$ | $(1,2,3,4,5,6,7,8)$ |

Determine an order in which these items should be processed so as to minimize the total processing time.

**Step** 1: Using robust ranking technique for octagonal fuzzy number, the fuzzy times can be converted in to crisp times.

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)d\alpha$$

$R(-2,-1,0,1,4,5,6,7) = 5$ $R(1,2,3,4,5,6,9,10) = 10$

$R(1,2,4,5,8,10,12,14) = 14$ $R(2,3,6,7,8,10,11,13) = 15$

$R(3,5,6,8,10,12,13,15) = 18$ $R(-3,-2,-1,0,1,2,3,4) = 1$

$R(-3,-2,-1,0,4,5,6,7) = 4$ $R(-4,-3,0,1,2,3,4,5) = 2$

$R(-2,-1,0,1,2,3,4,5) = 3$ $R(-1,0,1,2,3,4,5,6) = 5$

$R(7,8,10,11,14,15,17,18) = 25$ $R(6,8,9,10,12,13,14,16) = 17$

$R(-1,0,1,2,4,5,6,7) = 6$ $R(5,6,7,8,12,13,14,15) = 20$

$R(1,2,3,4,5,6,7,8) = 9$

| **Items** | **Cutting** $M_1$ | **Sewing** $M_2$ | **Pressing** $M_3$ |
|---|---|---|---|
| $A_1$ | 5 | 1 | 25 |
| $A_2$ | 10 | 4 | 17 |
| $A_3$ | 14 | 2 | 6 |
| $A_4$ | 15 | 3 | 20 |
| $A_5$ | 18 | 5 | 9 |

**Step (2)**: Three machine problem can be converted into two machine problem

| **Items** | **Machine** $M_1$ | **Machine** $M_2$ |
|---|---|---|
| $A_1$ | 6 | 26 |
| $A_2$ | 14 | 21 |
| $A_3$ | 16 | 8 |
| $A_4$ | 18 | 23 |
| $A_5$ | 23 | 14 |

∴ **Optimum sequence**: $A_1$, $A_2$, $A_4$, $A_5$, $A_3$

**Step (3)**: Total Elapsed time and Idle time

| Jobs | Machine - $M_1$ | | Machine - $M_2$ | | Machine - $M_3$ | |
|---|---|---|---|---|---|---|
| | Time in | Time out | Time in | Time out | Time in | Time out |
| $J_3$ | 0 | 5 | 5 | 6 | 6 | 31 |
| $J_1$ | 5 | 15 | 15 | 19 | 31 | 48 |
| $J_4$ | 15 | 30 | 30 | 33 | 48 | 68 |
| $J_5$ | 30 | 48 | 48 | 53 | 68 | 77 |
| $J_2$ | 48 | **62** | 62 | **64** | 77 | **83** |

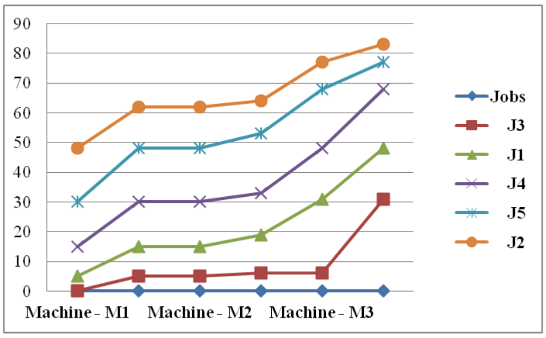


**Figure 2.** Graphical representation of total elapsed time and idel time

Using the above procedure the problem can be reduced as following optimal solution:
Total Elapsed time = 83Hrs
Idle time on Machine $M_1$ = 21 Hrs
Idle time on Machine $M_2$ = 49Hr

## 6. Conclusion

This paper has been presented the successful implementation of Octagonal Fuzzy Numbers (OFNs); this method has been provided simple and powerful ranking criteria of solving fuzzy sequencing problem. The fuzzy sequencing problem of OFNs has been transformed into crisp sequencing problem using robust ranking technique. We extended the well-known Johnson's algorithm to fuzzy numbers to identifying the order exclusive of changing them to conventional sequencing problem. By this method we obtained the optimal solution in fuzzy nature and the optimal total cost in crisp nature.

This new method is illustrated by solving numerical example. This method is very easy to understand and to apply for solving fuzzy sequencing problems in real life situation. This new concept of ranking method can be used to all types of sequencing problems which would give effective solutions for any uncertain data.

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